Indian J Phys. **82** (5), 621-625 (2008)

An analysis of self-similarity in quark and gluon densities at small x

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Abstract Sometimes back the concept of self-similarity in the structure of the proton at small x has been introduced We comment on the limitations of the models based on self similarity We then extend the formalism and phenomenology to unintegrated and integrated gluon densities

Keywords Self similarity fractal dimension deep inelastic scattering gluon density low x

PACS Nos 05-45 Df 47 53 +n 12 38 t 13 60 Hb

1. Introduction

Application of self-similarity [1,2] in hadron multiparticle production processes [3-10], diffractive DIS involving Pomeron exchange processes [11,12], quantum gravity [13,14] and quantum Regge calculus [15] are getting attention in recent years However the use of fractal geometry in the study of the proton structure itself has not yet attracted wider attention In this direction, a specific statistical quark model pursued in Ref [16] suggests that hadrons like proton are described by a self-similar object of fractal dimension 9/2 and anomalous dimension 3/2 Later in deep inelastic processes, a form of structure function incorporating self-similarity has been reported in Ref [17] In this approach the resultant expression contains four unknown parameters D_0, D_1, D_2 and D_3 to be determined from data While one of them is just the normalization constant, other three are identified as the fractal dimensions which are fitted to HERA collider data [18, 19] In a sense, fractality here is used as a tool to provide a parameterization involving a few parameters that are fitted to data The specific parameterization provides an excellent description of the data which covers a region of four momentum transfer squared $0.045 \leq Q^2 \leq 150$ GeV^2 and of Bjorken variable $6.2 \times 10^{-7} \leq x \leq 0.2$

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The formalism described in Ref. [17] is based on the imposition of self-similarity constraints to the dimensionless unintegrated quark density $q_i(x,k_T^2)$ and relating them to the integrated density from the relation,

$$q_i\left(x,Q^2\right) = \int^{Q^2} dk_T^2 q_i\left(x,k_T^2\right).$$

In eq. (1) k_T^2 represents the transverse momentum of the quark while Q^2 is the four momentum squared of the virtual photon. One of the fitted parameter D_3 was estimated to be negative ($D_3 \sim -1.2$). Questions were therefore raised [20,21] and an alternative parameterization of the structure function with an estimated positive value of D_3 was suggested recently [22].

Important quantities in the short distance behavior of the nucleons besides the structure function $F_2^p(x, Q^2)$ are the unintegrated and integrated gluon densities $xg(x,k_T^2)$ and $xg(x,Q^2)$ [23,24]. It is also well known that [25-27] in the case of $F_2(x,Q^2)$, the gluon distribution enters indirectly and determines the slope in $\ln(Q^2)$ of the structure function.

In the present paper, we will investigate the compatibility of gluon densities based on the notion of self-similarity with the approximate relations available in the literature [25-27] based on QCD evolution equation. We will also suggest their plausible modifications to accommodate the fractal models.

2. Formalism

2.1. Gluon density and self-similar models :

The forms of unintegrated and integrated gluon densities with self-similarity are identical to those of the quark distribution reported in Ref. [17] and [22] except the set of parameters will be different. Thus two alternative forms of unintegrated gluon density $g(x,k_T^2)$ are

$$\log g'\left(x,k_T^2\right) = D_1^g \log 1/x \log\left(\frac{Q_0^2 + k_T^2}{Q_0^2}\right) + D_2^g \log 1/x + D_3^g \log\left(\frac{Q_0^2 + k_T^2}{Q_0^2}\right) + D_0^g \quad (2)$$

and

$$\log g''\left(x,k_T^2\right) = D_1^g \log 1/x \log\left(\frac{Q_0^2}{Q_0^2 + k_T^2}\right) + D_2^g \log 1/x + D_3^g \log\left(\frac{Q_0^2}{Q_0^2 + k_T^2}\right) + D_0^g. \quad (3)$$

Using relation similar to eq. (1), eq (2) and (3) lead to two alternative forms of gluon density.

$$xg'(x,Q^2) = \frac{x^{-D_2^g+1}}{1 + D_3^g + D_1^g \log 1/x} \; {}^{-D_1^g\left(1+Q^2/Q_0^2\right.} \left. 1 + Q^2/Q_0^2 \right)^{D_3^g+1} - 1 \quad (4)$$

$$xg^{II}(x,Q^2)=\frac{e^{D_0^g}Q_0^2 x^{-D_2^g+1}}{1-D_3^g-D_1^g\log\frac{1}{x}}\left[x^{D_1^g\left(1+Q^2/Q_0^2\right)}\left(1+\frac{Q^2}{Q_0^2}\right)^{D_3^g+1}-1\right] \tag{5}$$

corresponding to the models of Ref. [17] and [22] respectively.

A comparison of eqs. (4) and (5) with exact results on gluon density [27] will need four parameters D_0^g, D_1^g, D_2^g and D_3^g to be determined.

But gluon densities are not directly measurable- they are measured from the slope of the structure function $\partial F_2/\partial\log Q^2$ through DGLAP equations based relation [25]

$$G\left(2x,Q^2\right)=\frac{9\pi}{5\alpha_S}\frac{3}{2}\frac{\partial F_2\left(x,Q^2\right)}{\partial\log Q^2}. \tag{6}$$

The relation (6) has later been improved to [26]

$$G\left(\frac{4}{3}x\right)\approx\frac{3\pi}{5\alpha_S}\frac{\partial F_2\left(x,Q^2\right)}{\partial\log Q^2}. \tag{7}$$

In our analysis we will investigate the compatibility of relation (eqs. (6)-(7)) with self-similarity and suggest plausible empirical modifications besides direct fitting of eq. (4) and (5) to exact results [27].

3. Results and discussion

We choose MRST04LO[27] solutions for comparison. A comparison of eq. (4) and (5) with MRST04LO[27] exact results gives the following set of parameters,

$D_0^g = 0.339$, $D_1^g = 0.073$, $D_2^g = 1.5011$, $D_3^g = -1.287$, $Q_0^2 = 0.062$ for eq. (4),

$D_0^g = 2.5597$, $D_1^g = 0.239$, $D_2^g = 1.5066$, $D_3^g = 1.435$, $Q_0^2 = 0.049$ for eq. (5),

where we see that the values of D_0 and D_2 for gluons is different than that of quarks as obtained in Ref.[17,22]. Regarding the normalization constant D_0, it may be different for gluons since it has a flavor dependence. Regarding D_2, we observe that $D_2^g > D_2^q$, which conforms to the feature that for any Q^2, gluon is steeper than quark density. In Figure (1) we have plotted the variation of $xg(x,Q^2)$ with x for various Q^2. We observe that our prediction lies below the exact results for $Q^2>50\ GeV^2$. Regarding the formalism of Lastovicka, it has certain crossover point for all the Q^2 values. Above the crossover point the result of Lastovicka lies above the exact results and below it, it lies below the exact results.

Let us now discuss the compatibility of our results with the DGLAP based relations (6)-(7) Our preliminary investigation shows that indeed the gluon is related to the slope of the structure function but the relation between them don't obey (6)-(7). They are slightly modified Empirically we have found, eqs. (6) and (7) are modified to

$$G\left(2x, Q^2\right) = \frac{9\pi}{5\alpha_S}\frac{3}{2}\frac{\partial F_2\left(x, Q^2\right)}{\partial \log Q^2} f(x) \tag{8}$$

with $f(x) = x^{0.1404}$ and $f(x) = x^{0.4748}$ corresponding to eq. (4) and (5) respectively.

A similar modification of eq. (7) is,

$$G\left(\tfrac{4}{3}x\right) \approx \frac{3\pi}{5\alpha_S}\frac{\partial F_2\left(x, Q^2\right)}{\partial \log Q^2} f(x) \tag{9}$$

with $f(x) = x^{0.457}$ and $f(x) = x^{0.6434}$ corresponding to eqs. (4) and (5) respectively

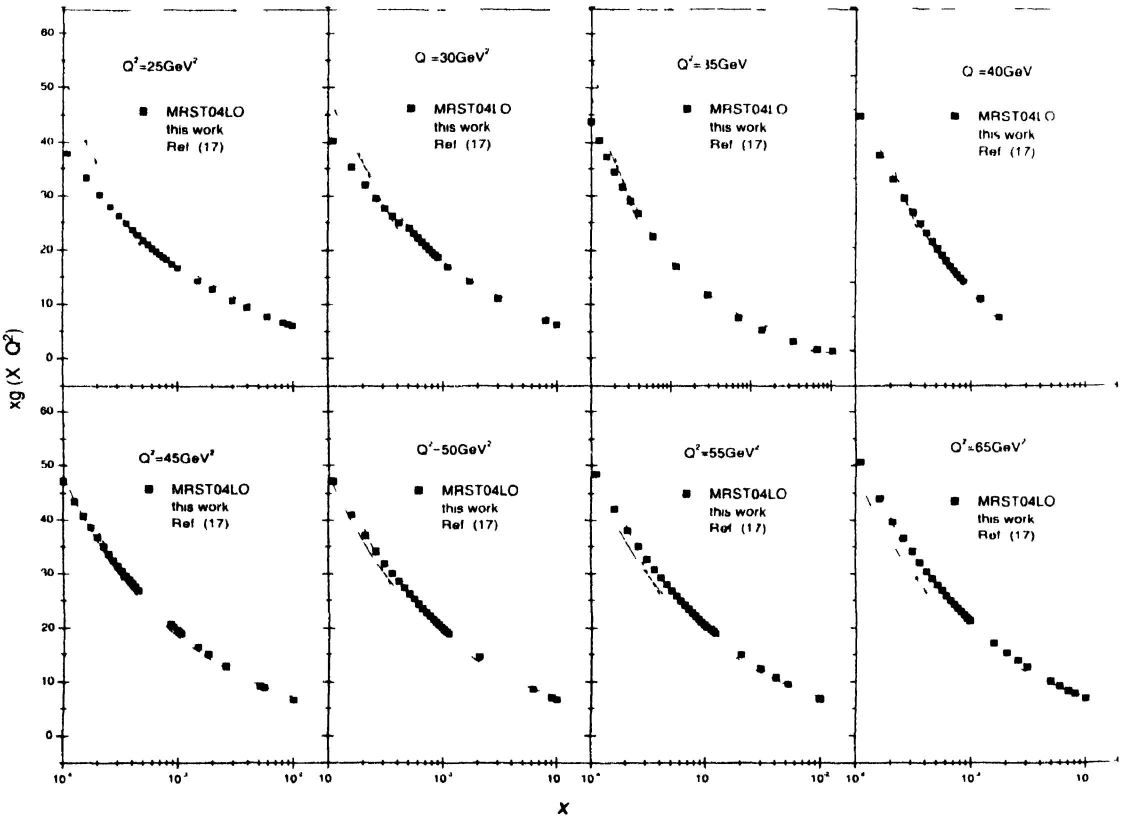

Figure 1. $xg(x,Q^2)$ as a function of x in bins of Q^2

Acknowledgment

One of the authors (Rupjyoti Gogoi) is thankful to Council of Scientific & Industrial Research, New Delhi for financial support.

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